

Multiscale Model Diagnostic

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Model diagnostic of prior predictive distribution

... prior predictive distribution

$$x_e = g(m_e) + \epsilon_e; \quad e = 1, \dots, E$$

g : forward model/simulator

m_e : prior parameter realization

ϵ_e : error realization

x_e : (perturbed) prior prediction

Model diagnostic of prior predictive distribution

...to reduce the risk of unsuccessful data assimilation

Assess if prior predictions are consistent with observed data, that is, if the observed data vector could be a credible realization from the prior predictive distribution

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If not consistent, (part of) the modeling setup should be changed before data assimilation is performed

Model diagnostic of prior predictive distribution

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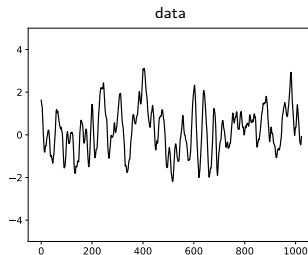
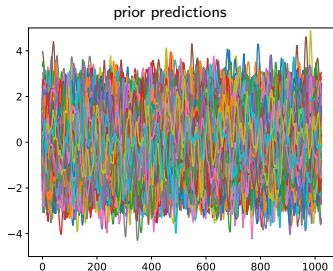
Assess if prior predictions are consistent with observed data, that is, if the observed data vector could be a credible realization from the prior predictive distribution

If not consistent, (part of) the modeling setup should be changed before data assimilation is performed

Coverage of individual observations by the ensemble of prior predictions is not sufficient for consistency, as coverage does not take into account trends and shapes

Model diagnostic of prior predictive distribution

The task



Could the black curve be a credible realization from the prior predictive distribution?

Multiscale model diagnostic of prior predictive distribution

Compare prior predictions and data on multiple scales of variation

Multiscale model diagnostic of prior predictive distribution

Compare prior predictions and data on multiple scales of variation

The method utilizes scalar products of prior predictions and data with certain multiscale vectors

$$\begin{aligned}\chi_{k,e} &= h_k^T x_e \\ \delta_k &= h_k^T d\end{aligned}$$

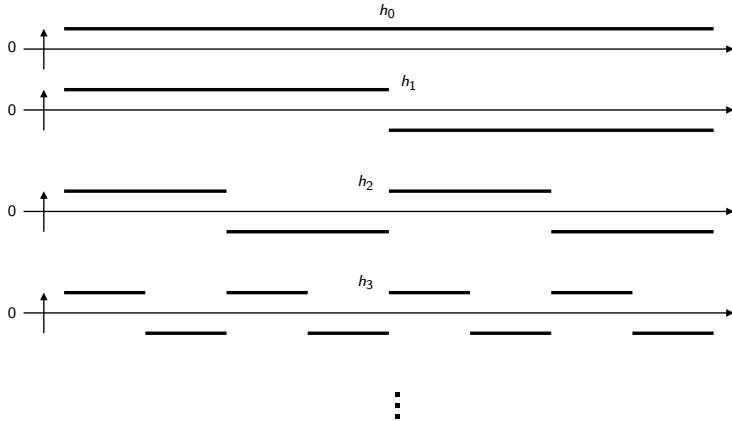
x_e : (recall) prior prediction

d : data

h_k : multiscale vector on scale k

Multiscale model diagnostic of prior predictive distribution

Multiscale vectors (the four vectors with the longest characteristic length)



Multiscale model diagnostic of prior predictive distribution

Application to real data

With real data, there is only a single data vector available

$$\delta_k = h_k^T d$$

Multiscale model diagnostic of prior predictive distribution

Application to real data and method assessment

With real data, there is only a single data vector available

$$\delta_k = h_k^T d$$

When assessing the method's applicability and robustness on toy problems, I will consider an ensemble of data realizations to avoid effects associated with a particular data vector

$$\delta_{k,e} = h_k^T d_e$$

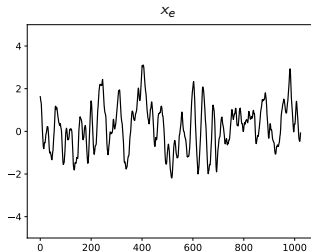
Empirical means, $M(\delta_k)$, and standard deviations, $S(\delta_k)$, will then be available for comparison with $M(\chi_k)$ and $S(\chi_k)$

Multiscale model diagnostic of prior predictive distribution

Method assessment on toy problems

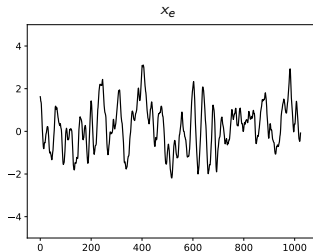
Method assessment on toy problems

Plot explanation



Method assessment on toy problems

Plot explanation



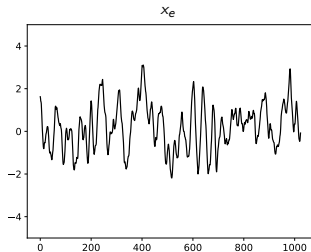
multiply x_e with

h_0 $\xrightarrow{\hspace{1.5cm}}$

to obtain $\chi_{0,e}$

Method assessment on toy problems

Plot explanation



multiply x_e with
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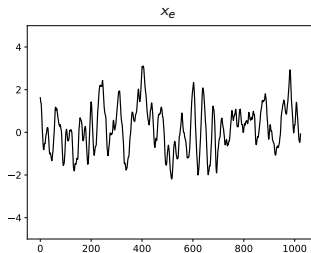
↓

Repeat for all e and compute

$M(\chi_0)$ and $S(\chi_0)$

Method assessment on toy problems

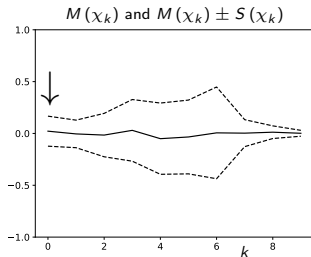
Plot explanation



→

multiply x_e with
 h_0 $\xrightarrow{\hspace{2cm}}$
to obtain $\chi_{0,e}$

↓

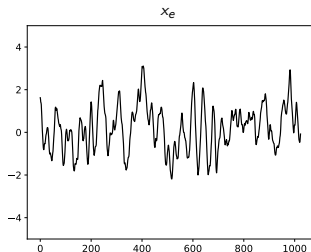


←

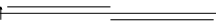
Repeat for all e and compute
 $M(x_0)$ and $S(x_0)$

Method assessment on toy problems

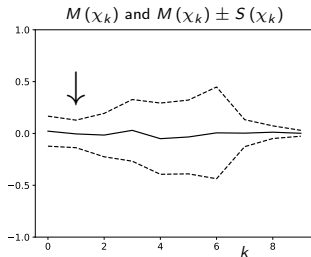
Plot explanation



→

multiply x_e with
 h_1 
to obtain $\chi_{1,e}$

↓



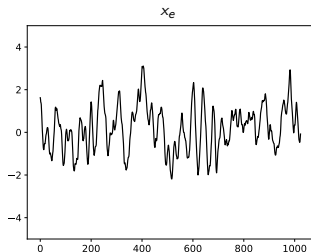
←

Repeat for all e and compute


$M(\chi_1)$ and $S(\chi_1)$

Method assessment on toy problems

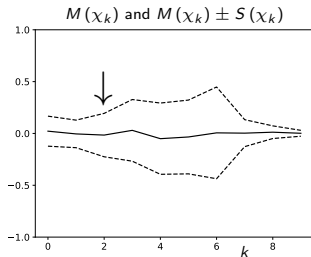
Plot explanation



→

multiply x_e with
 h_2 
to obtain $\chi_{2,e}$

↓



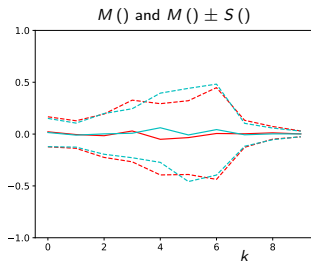
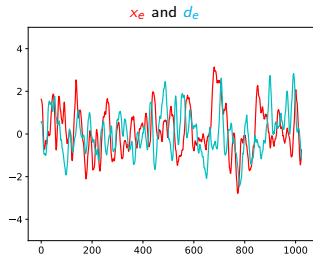
←

Repeat for all e and compute

$M(\chi_2)$ and $S(\chi_2)$

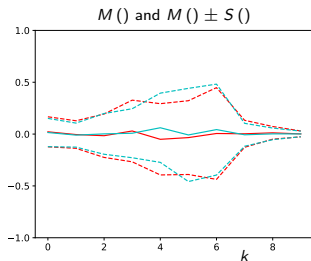
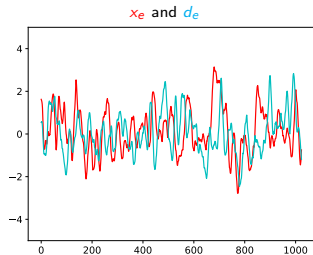
Method assessment on toy problems

Plot explanation, use of colors: prior predictive, data



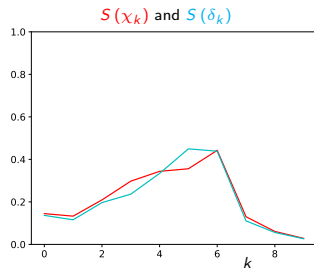
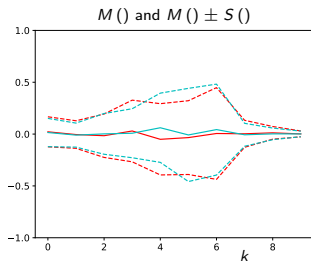
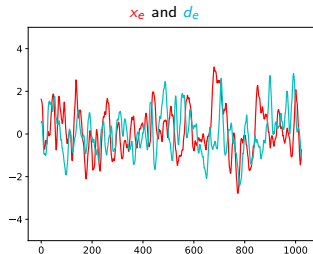
Method assessment on toy problems

Example 1 - x_e and d_e from the same distribution



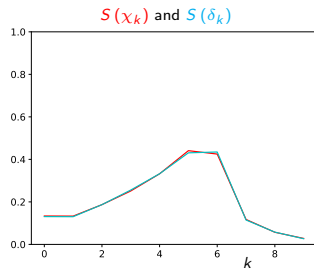
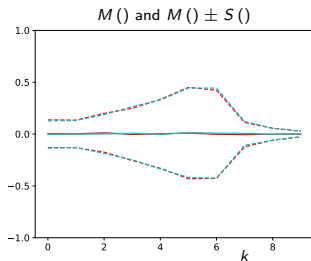
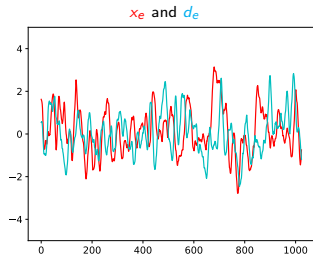
Method assessment on toy problems

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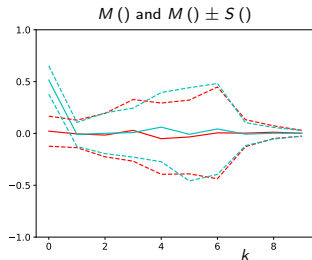
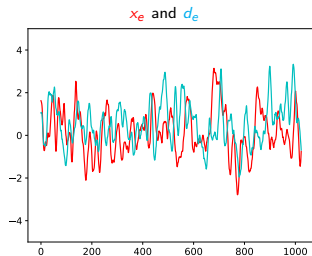
Method assessment on toy problems

Example 1 - x_e and d_e from the same distribution with ensemble size 1000



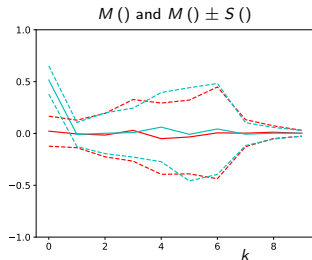
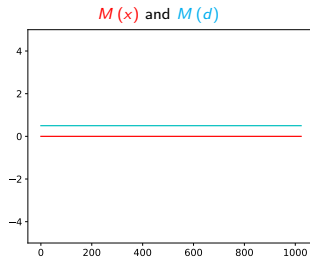
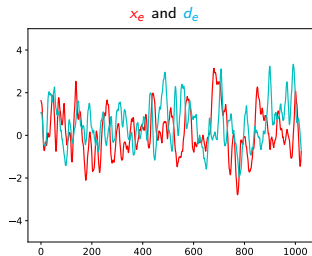
Method assessment on toy problems

Example 2



Method assessment on toy problems

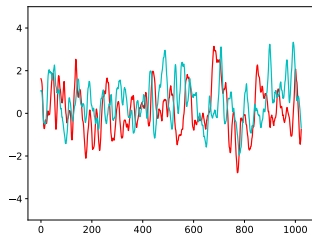
Example 2 - different data means



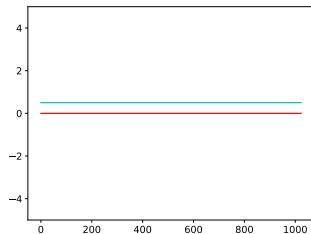
Method assessment on toy problems

Example 2 - different data means

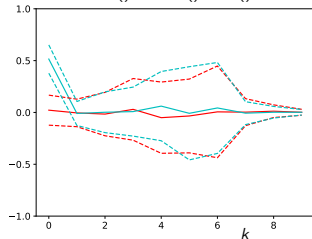
x_e and d_e



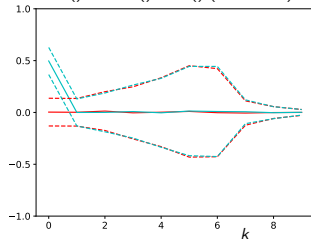
$M(x)$ and $M(d)$



$M()$ and $M() \pm S()$

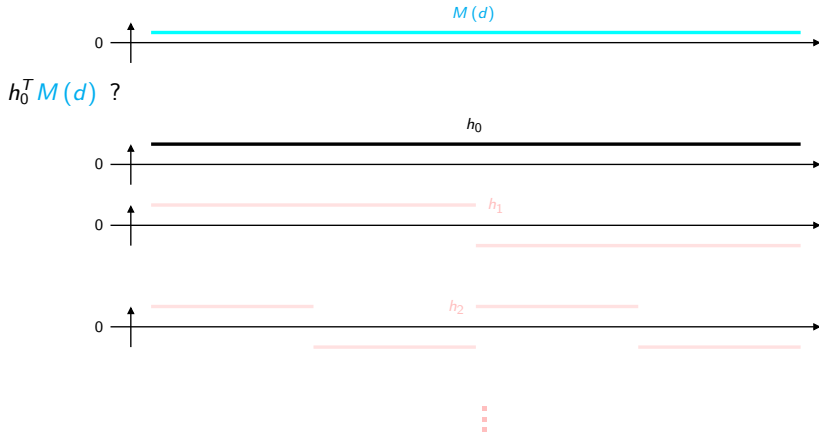


$M()$ and $M() \pm S()$ ($E = 1000$)



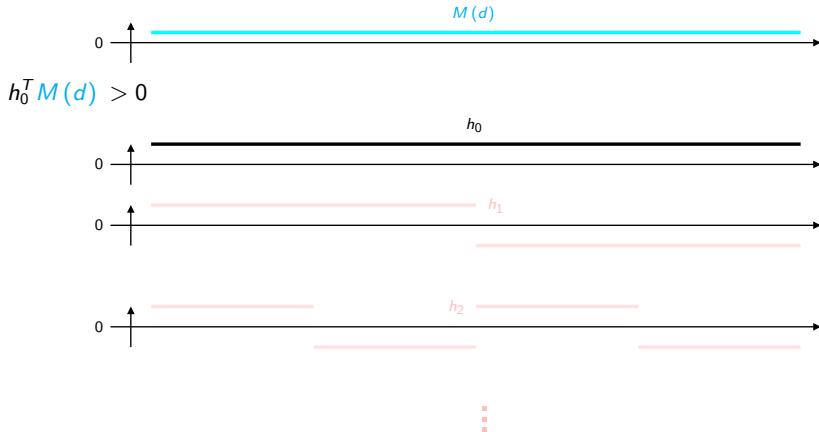
Method assessment on toy problems

Example 2 - simplified explanation of behaviour



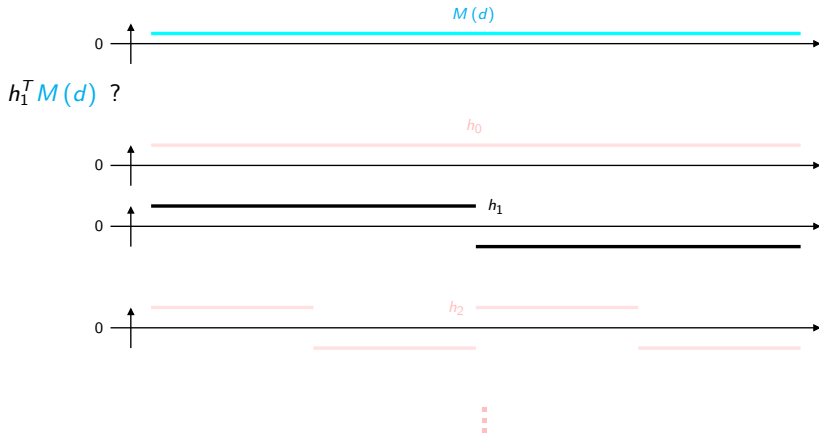
Method assessment on toy problems

Example 2 - simplified explanation of behaviour



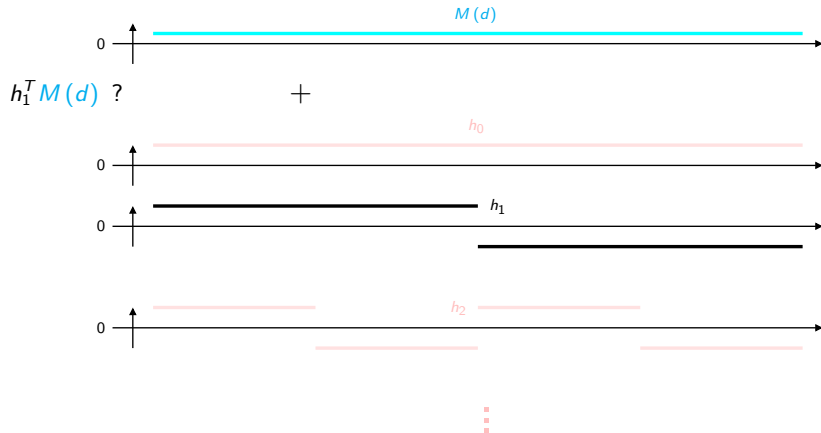
Method assessment on toy problems

Example 2 - simplified explanation of behaviour



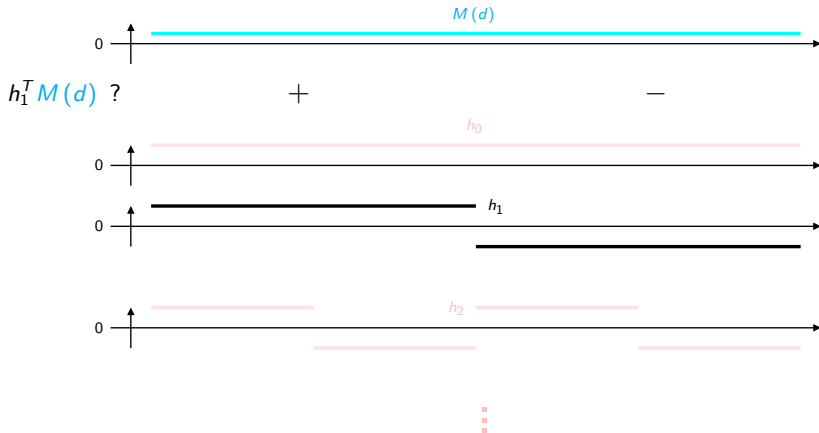
Method assessment on toy problems

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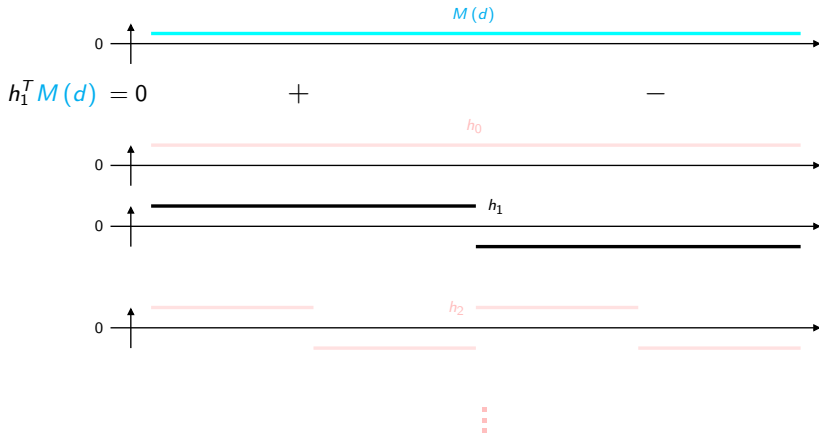
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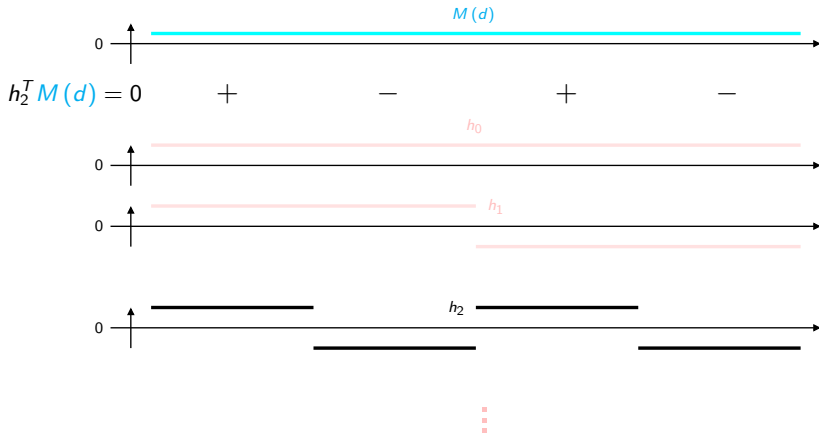
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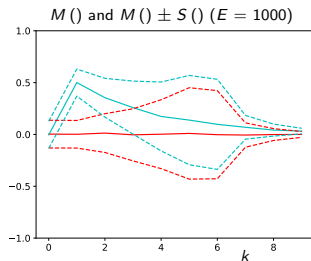
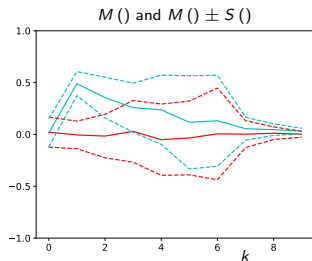
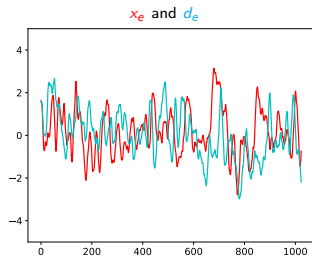
Method assessment on toy problems

Example 2 - simplified explanation of behaviour



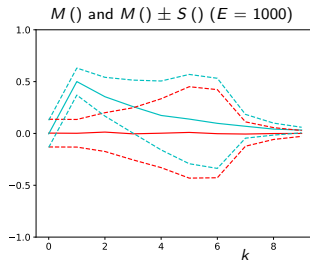
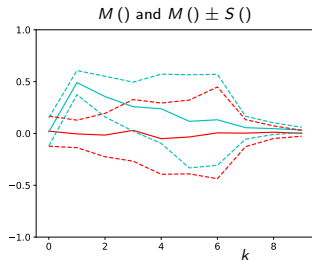
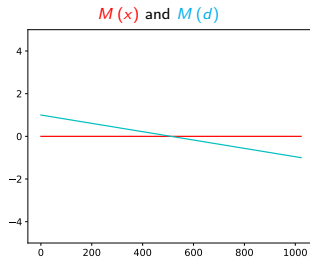
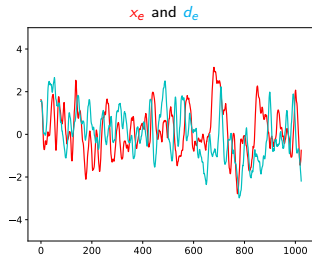
Method assessment on toy problems

Example 3



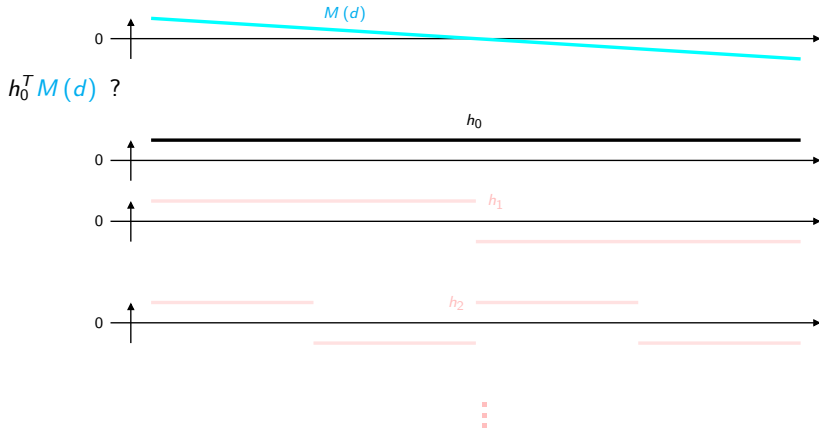
Method assessment on toy problems

Example 3 - decreasing data mean



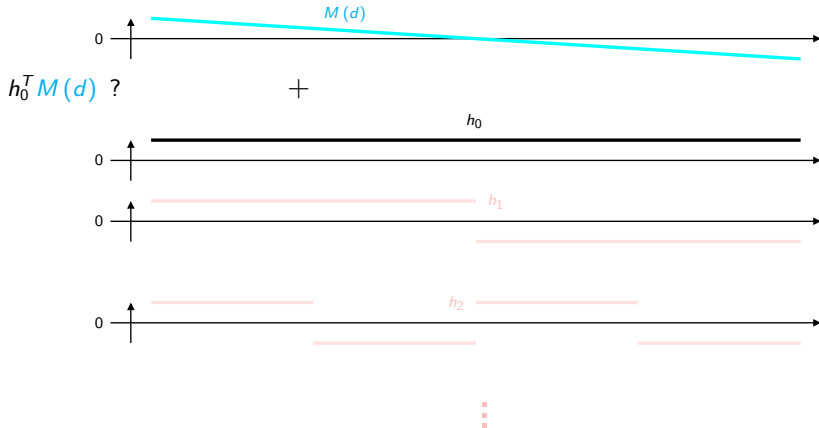
Method assessment on toy problems

Example 3 - simplified explanation of behaviour



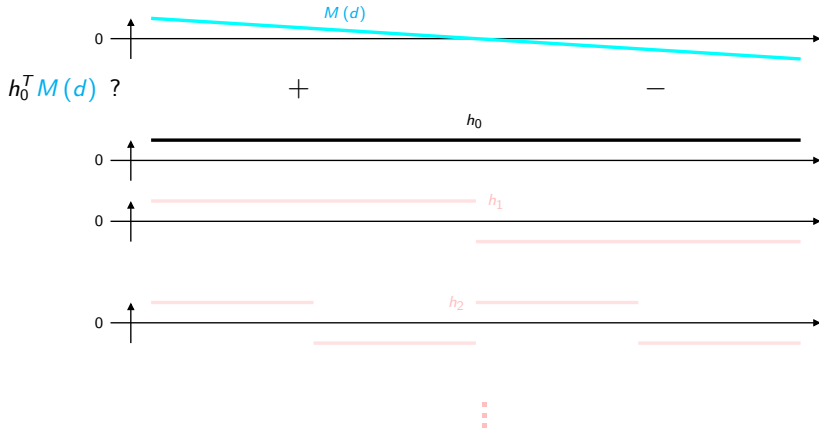
Method assessment on toy problems

Example 3 - simplified explanation of behaviour



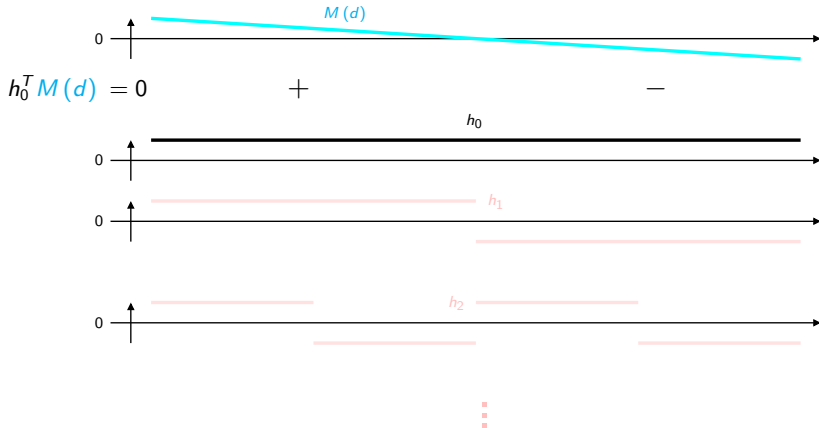
Method assessment on toy problems

Example 3 - simplified explanation of behaviour



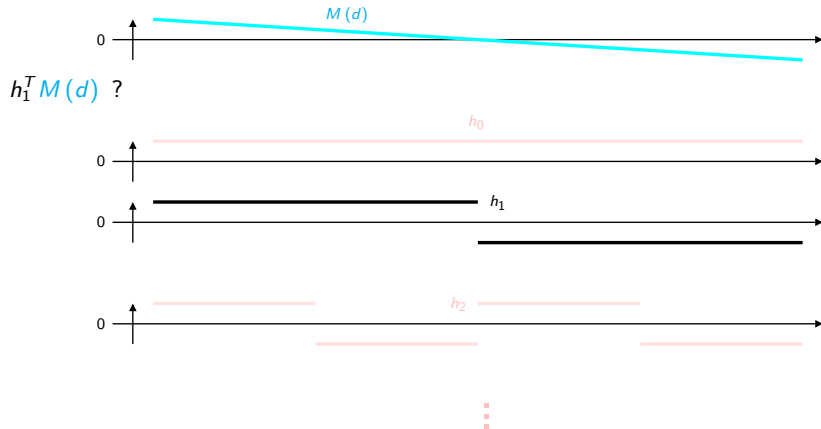
Method assessment on toy problems

Example 3 - simplified explanation of behaviour



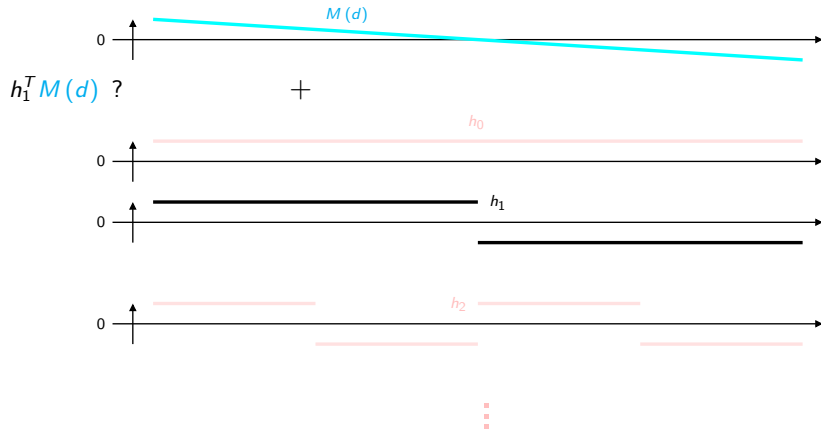
Method assessment on toy problems

Example 3 - simplified explanation of behaviour



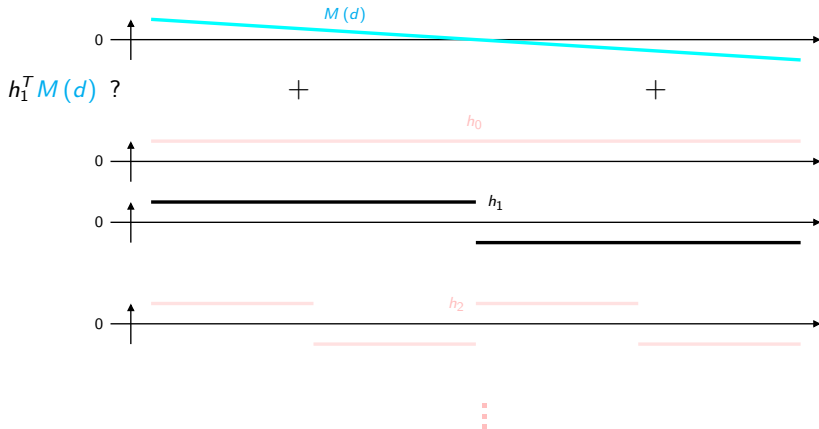
Method assessment on toy problems

Example 3 - simplified explanation of behaviour



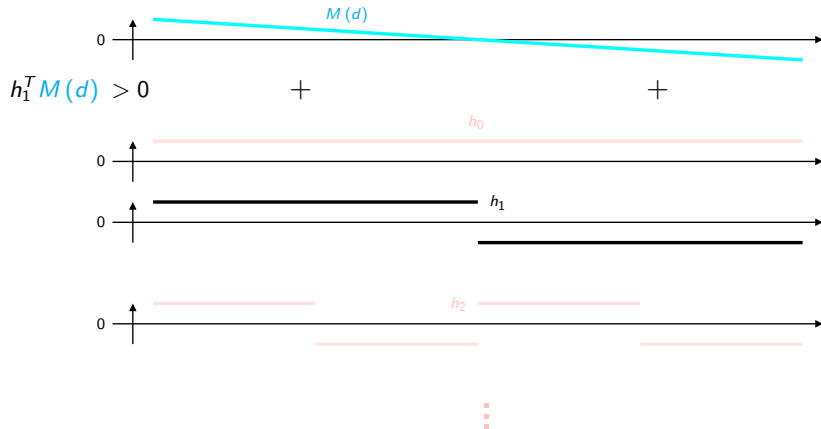
Method assessment on toy problems

Example 3 - simplified explanation of behaviour



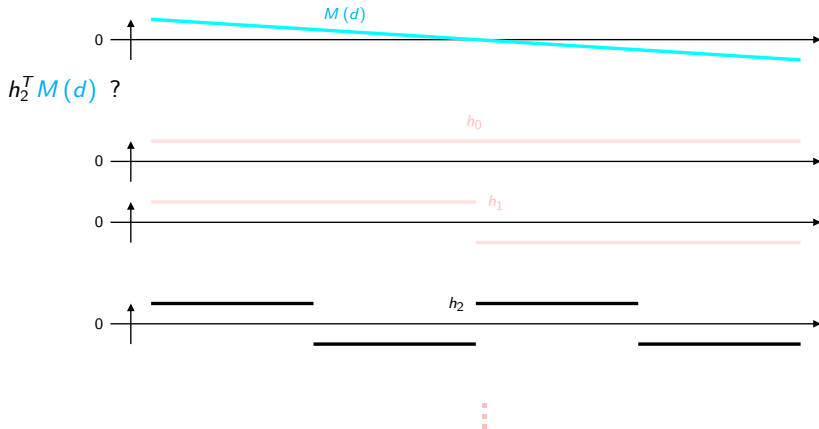
Method assessment on toy problems

Example 3 - simplified explanation of behaviour



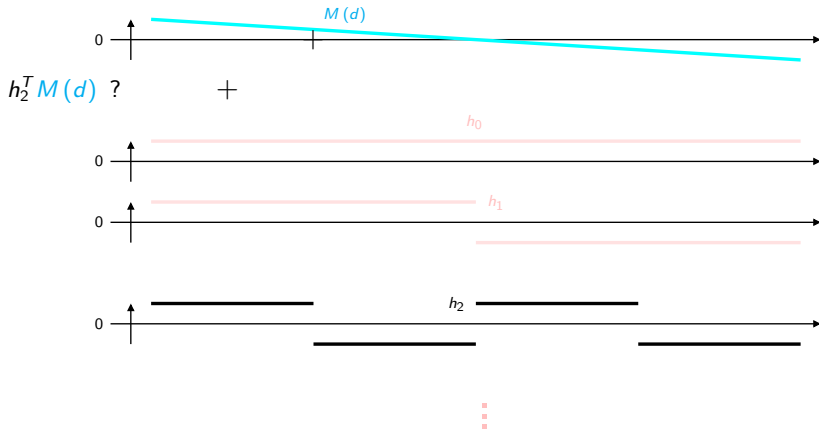
Method assessment on toy problems

Example 3 - simplified explanation of behaviour



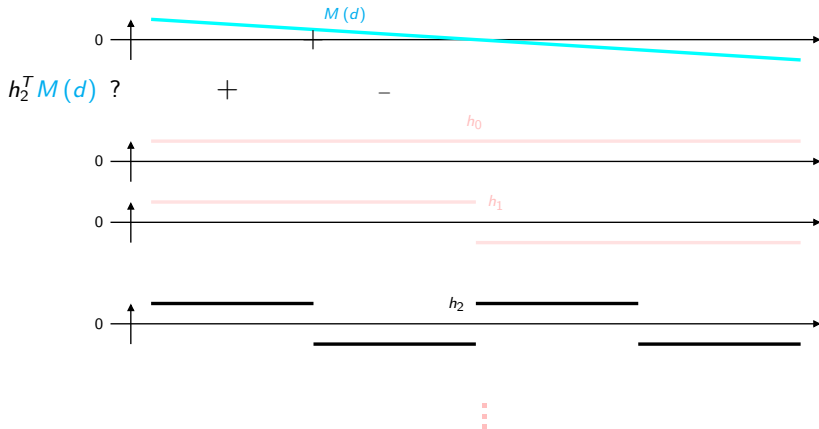
Method assessment on toy problems

Example 3 - simplified explanation of behaviour



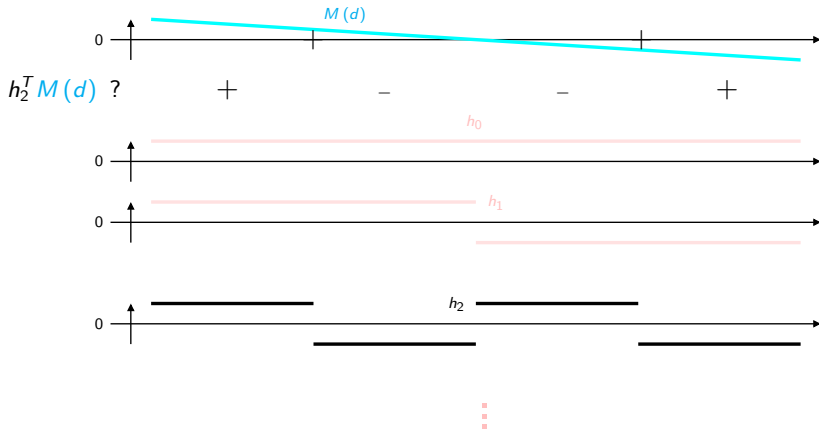
Method assessment on toy problems

Example 3 - simplified explanation of behaviour



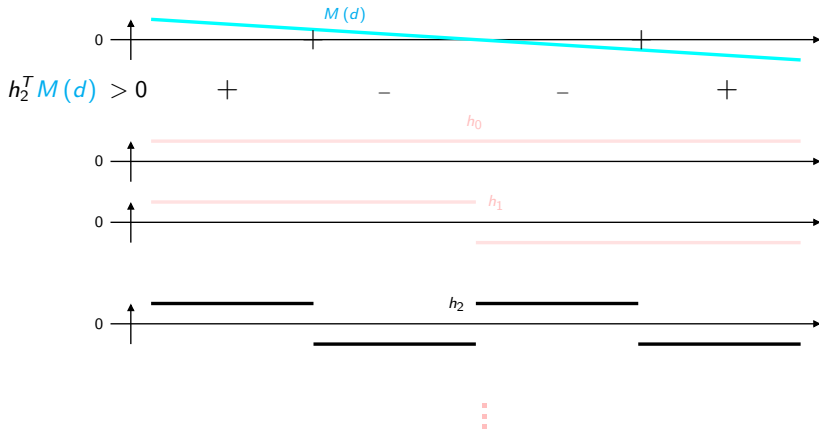
Method assessment on toy problems

Example 3 - simplified explanation of behaviour



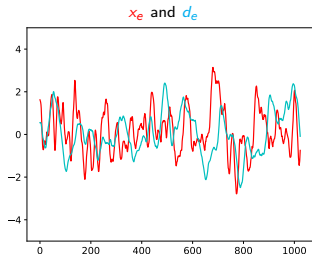
Method assessment on toy problems

Example 3 - simplified explanation of behaviour



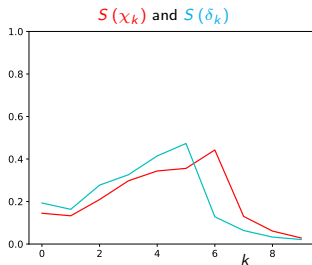
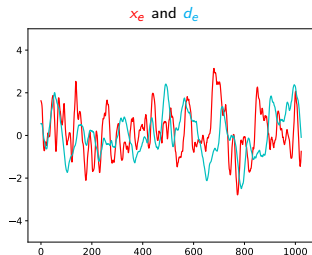
Method assessment on toy problems

Example 4 - correlation lengths 25 (x) and 50 (d)



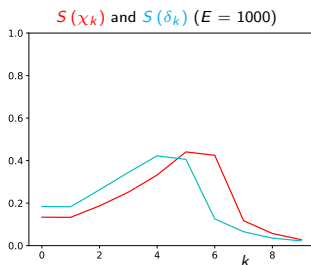
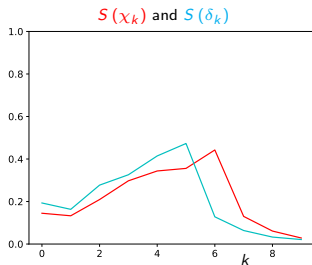
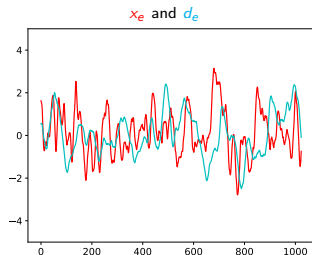
Method assessment on toy problems

Example 4 - correlation lengths 25 (x) and 50 (d)



Method assessment on toy problems

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Method assessment on toy problems

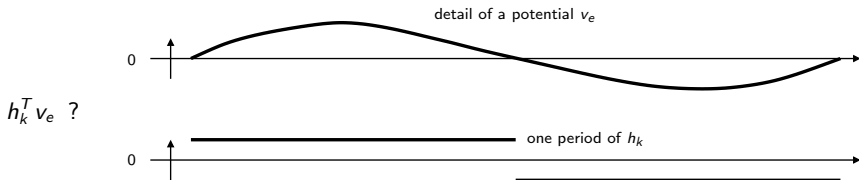
Example 4 - simplified explanation of behaviour

Let v denote either x or d

Method assessment on toy problems

Example 4 - simplified explanation of behaviour

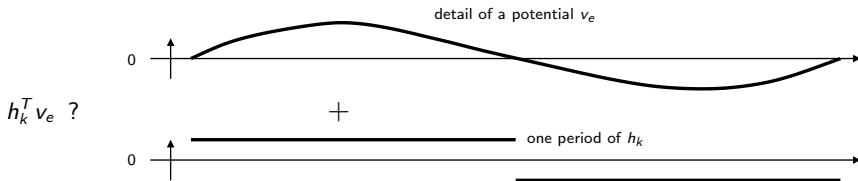
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Method assessment on toy problems

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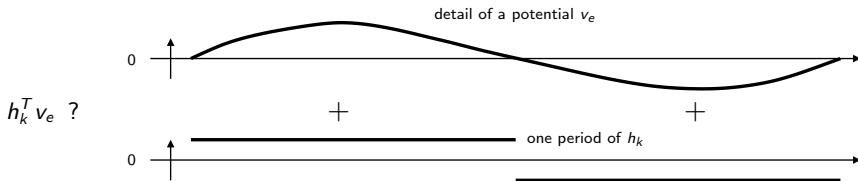
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Method assessment on toy problems

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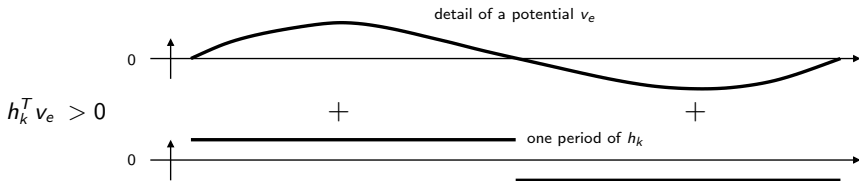
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Method assessment on toy problems

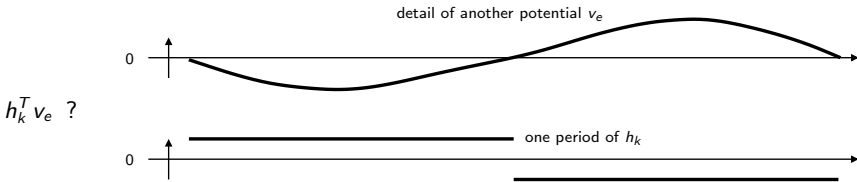
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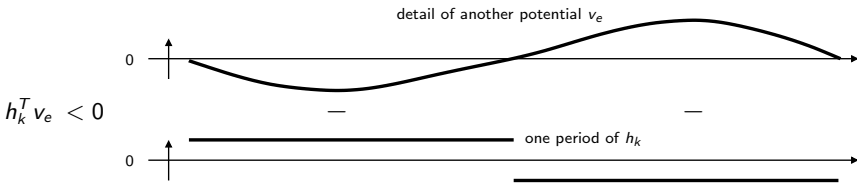
Method assessment on toy problems

Example 4 - simplified explanation of behaviour



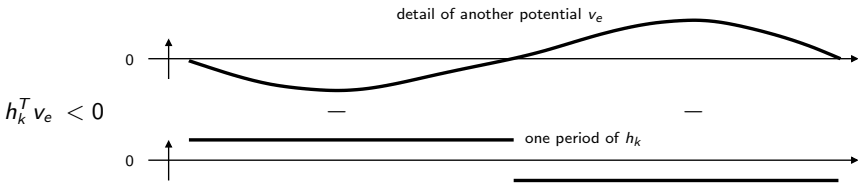
Method assessment on toy problems

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Method assessment on toy problems

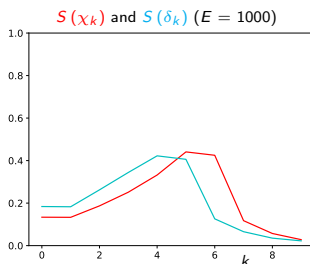
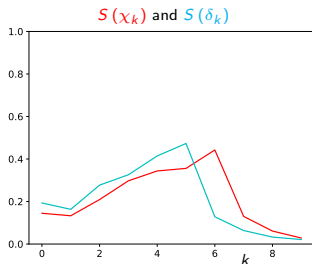
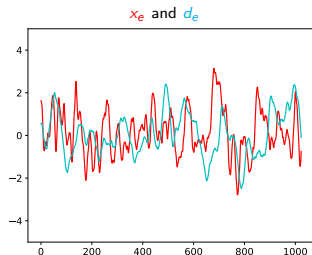
Example 4 - simplified explanation of behaviour



Large standard deviation of $h_k^T v_e$ when characteristic lengths of v_e and h_k are similar

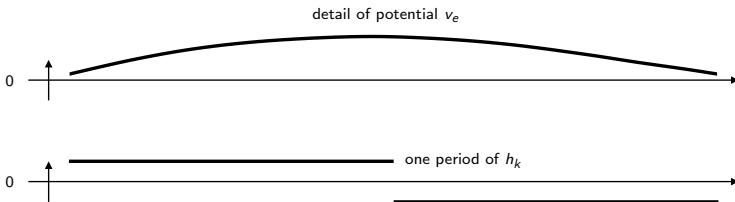
Method assessment on toy problems

Example 4 - correlation lengths 25 (x) and 50 (d)



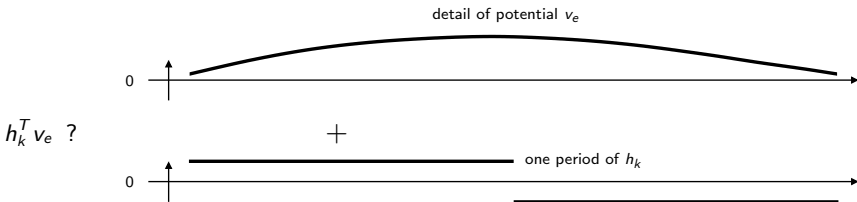
Method assessment on toy problems

Example 4 - simplified explanation of behaviour



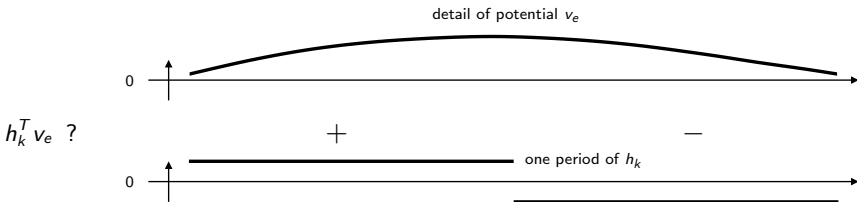
Method assessment on toy problems

Example 4 - simplified explanation of behaviour



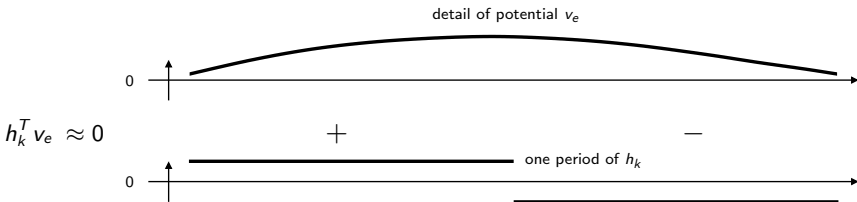
Method assessment on toy problems

Example 4 - simplified explanation of behaviour



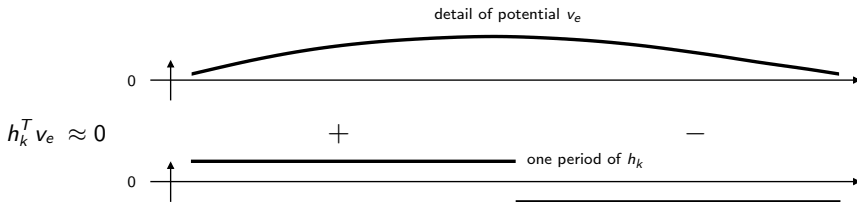
Method assessment on toy problems

Example 4 - simplified explanation of behaviour



Method assessment on toy problems

Example 4 - simplified explanation of behaviour



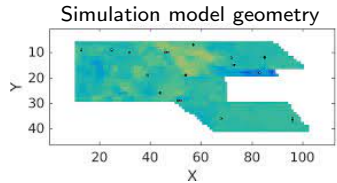
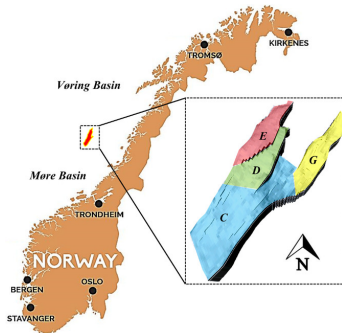
Small standard deviation when characteristic length of v_e is larger than that of h_k

Multiscale model diagnostic of prior predictive distribution

Application to real data

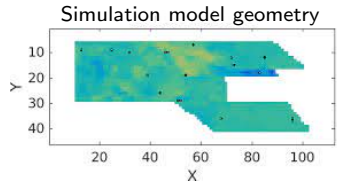
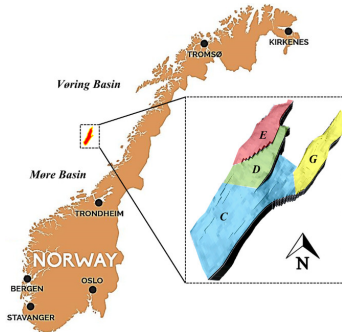
Application to real data

The Norne field



Application to real data

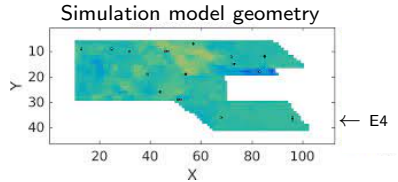
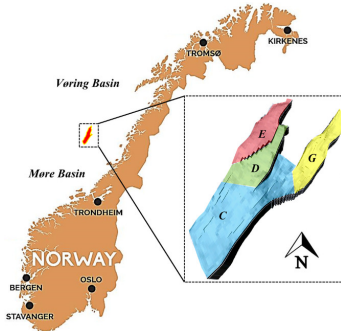
The Norne field



Data from G segment

Application to real data

The Norne field

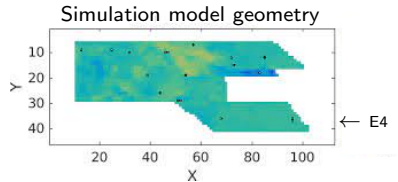
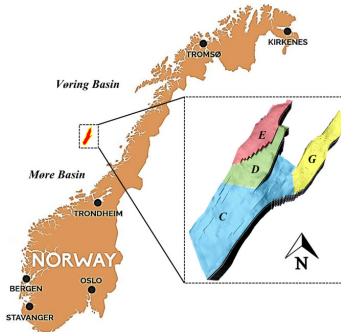


Data from G segment

Production data from well E4 (wells: black dots on right figure)

Application to real data

The Norne field



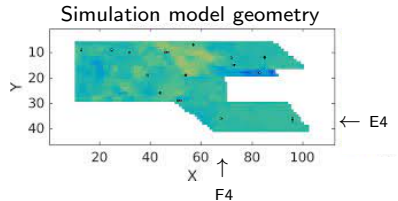
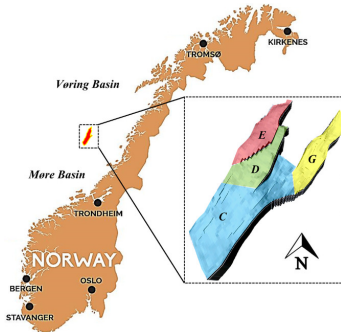
Data from G segment

Production data from well E4 (wells: black dots on right figure)

Time-lapse (4-D) impedance data from most of the segment

Application to real data

The Norne field



Data from G segment

Production data from well E4 (wells: black dots on right figure)

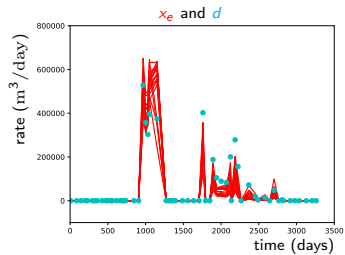
Time-lapse (4-D) impedance data from most of the segment

RFT data from wells E4 and F4

Application to real data

Production data - results for well E4

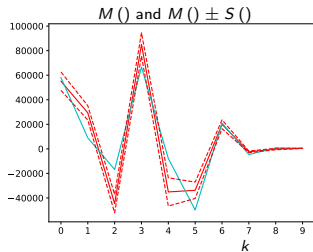
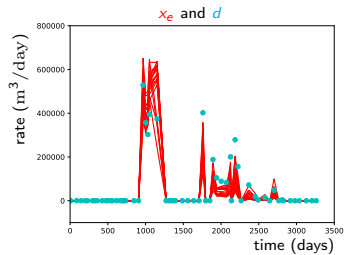
Gas



Application to real data

Production data - results for well E4

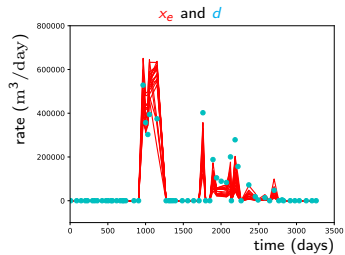
Gas



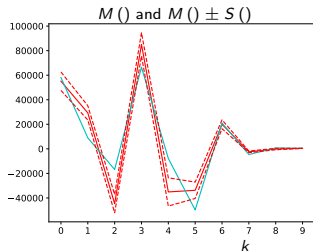
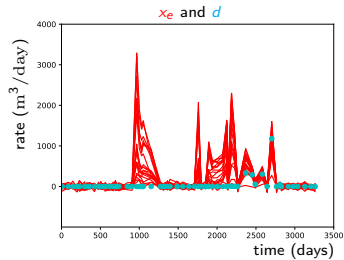
Application to real data

Production data - results for well E4

Gas



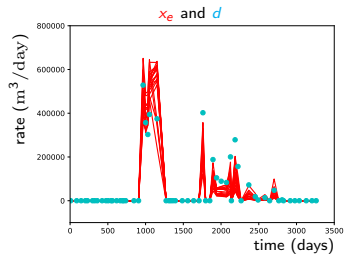
Water



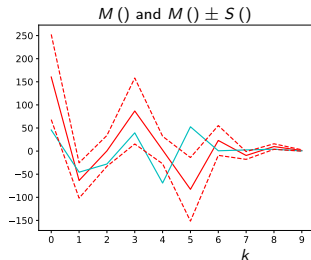
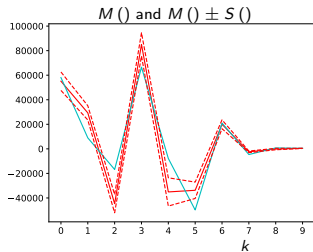
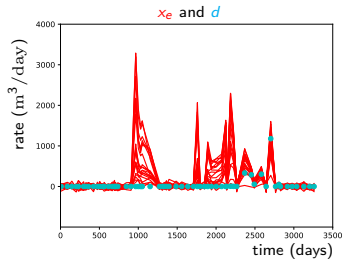
Application to real data

Production data - results for well E4

Gas



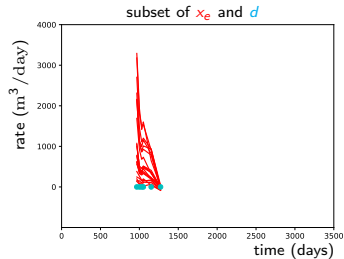
Water



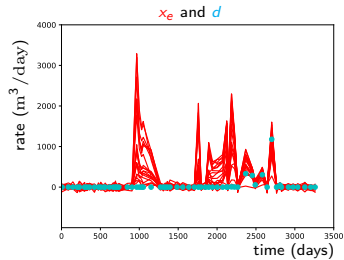
Application to real data

Production data - results for well E4

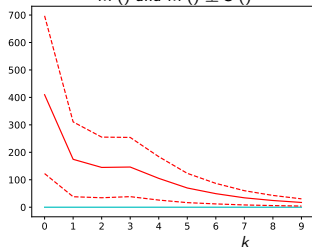
Water



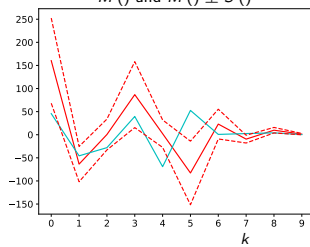
Water



$M()$ and $M() \pm S()$



$M()$ and $M() \pm S()$



Application to real data

Time-lapse impedance data

Time-lapse data are obtained by subtracting data acquired from two surveys over the same study region at different times. The aim is to infer fluid movements and/or pressure changes in the subsurface over this time span, and also to infer flow-related rock properties, such as permeability (fluid conductivity)

Application to real data

Time-lapse impedance data

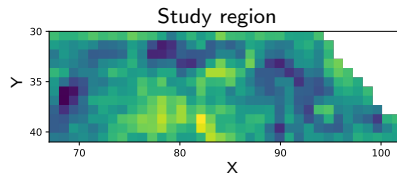
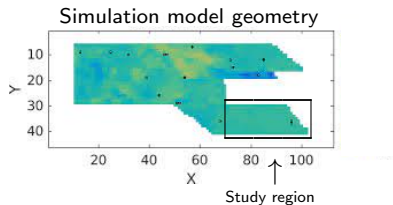
Time-lapse data are obtained by subtracting data acquired from two surveys over the same study region at different times. The aim is to infer fluid movements and/or pressure changes in the subsurface over this time span, and also to infer flow-related rock properties, such as permeability (fluid conductivity)

Impedance (density \times velocity) data for a subsurface region are obtained by inverting seismic data observed in the sea water or at the sea floor. Hence, they are not really data, but it is common to split the assimilation of time-lapse seismic data into flow-related rock properties this way

Application to real data

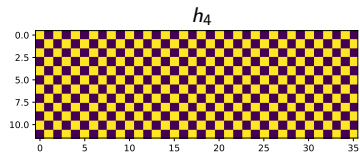
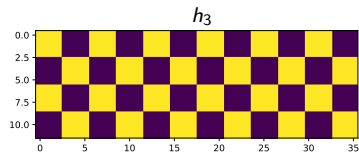
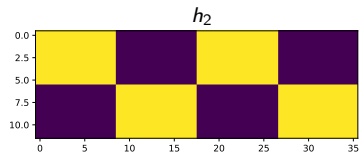
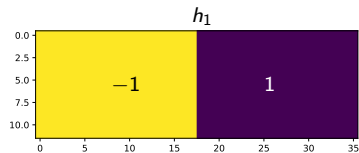
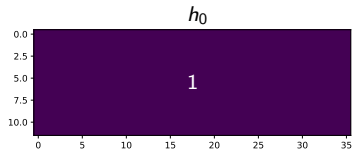
Time-lapse impedance data - study region

The major part of the Norne G segment constitutes the study region



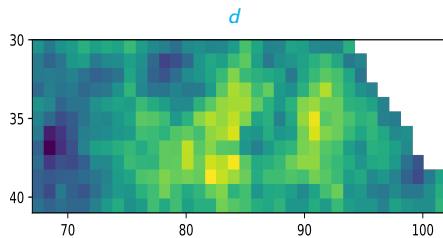
Application to real data

Time-lapse impedance data - multiscale vectors



Application to real data

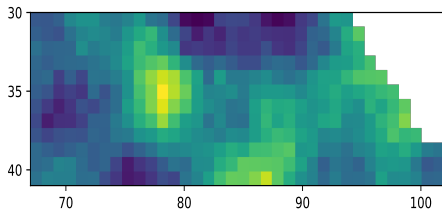
Time-lapse impedance data - time span 2001-2003



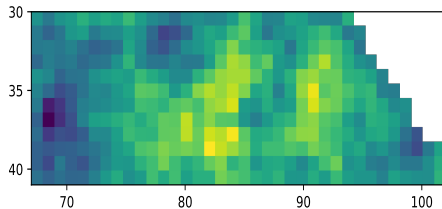
Application to real data

Time-lapse impedance data - time span 2001-2003

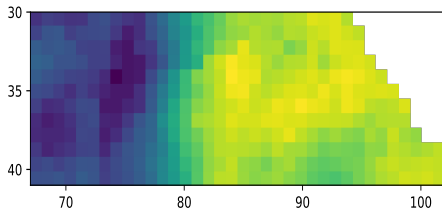
x_1



d

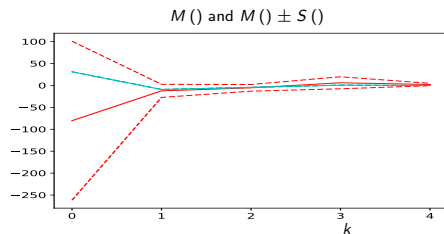
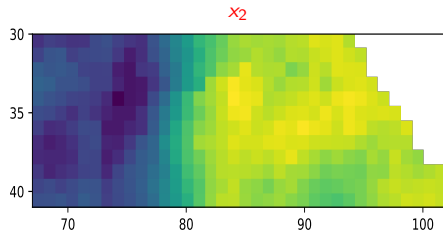
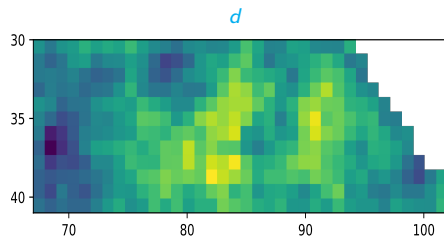
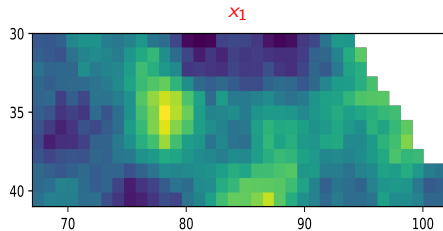


x_2



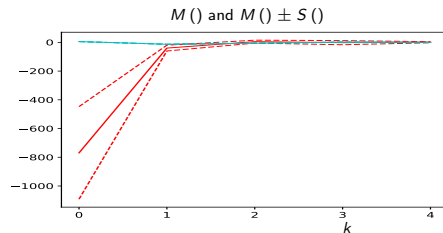
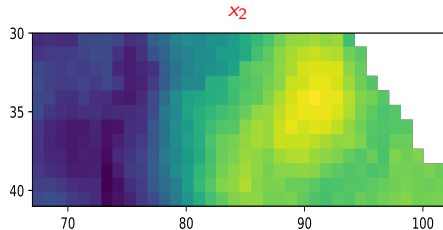
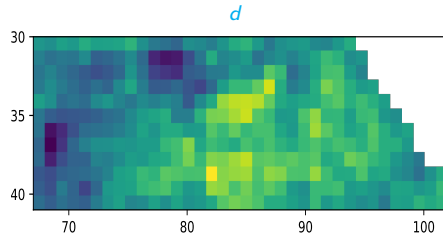
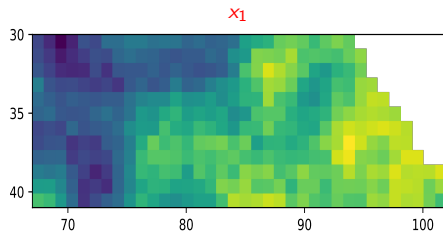
Application to real data

Time-lapse impedance data - time span 2001-2003 - results



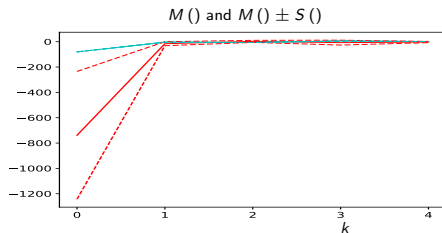
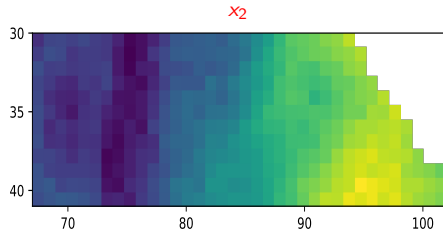
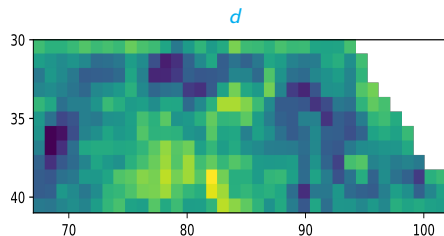
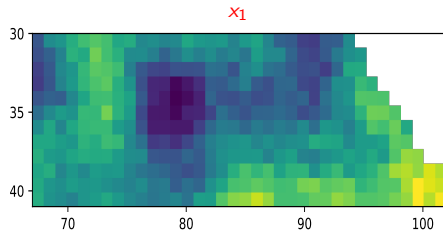
Application to real data

Time-lapse impedance data - time span 2001-2004 - results



Application to real data

Time-lapse impedance data - time span 2001-2006 - results

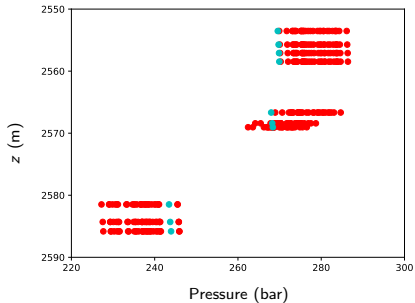


Application to real data

RFT data

RFT (Repeat Formation Tester) data consist of pressure values along the wellbore

Well E4

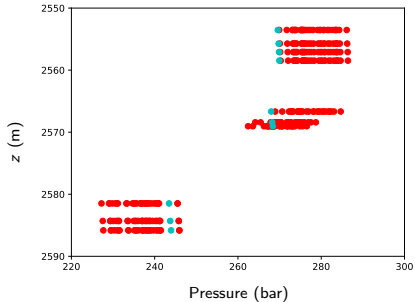


Application to real data

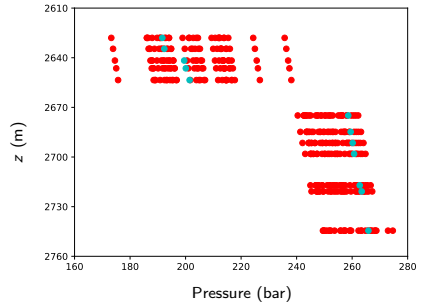
RFT data

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Well E4

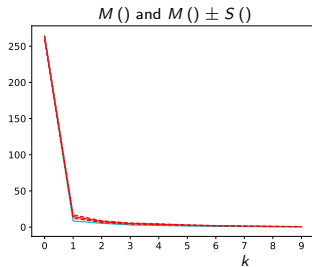
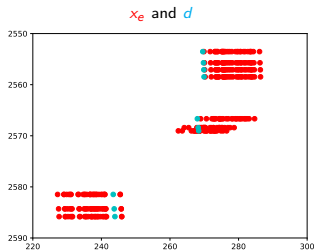


Well F4



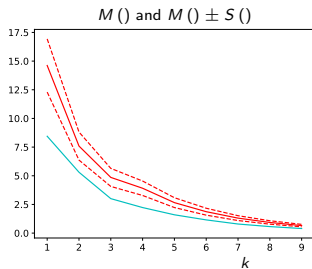
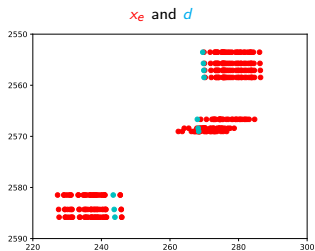
Application to real data

RFT data - results for well E4



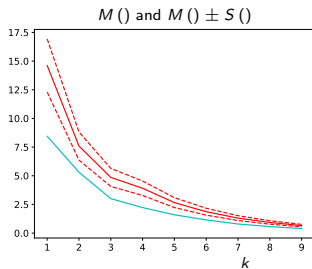
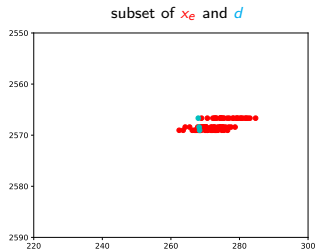
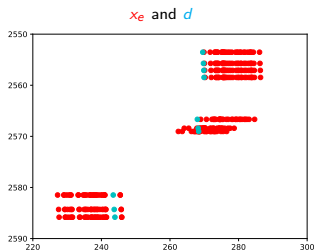
Application to real data

RFT data - results for well E4



Application to real data

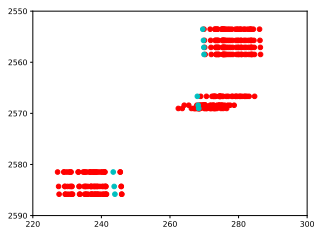
RFT data - results for well E4



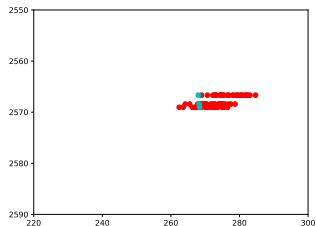
Application to real data

RFT data - results for well E4

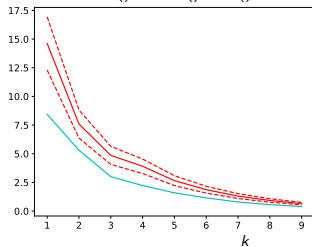
x_e and d



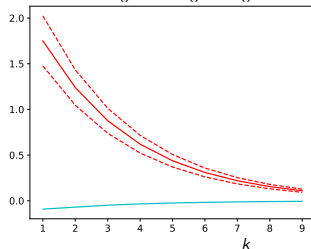
subset of x_e and d



$M()$ and $M() \pm S()$

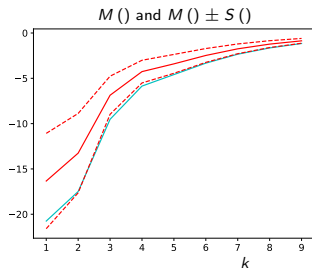
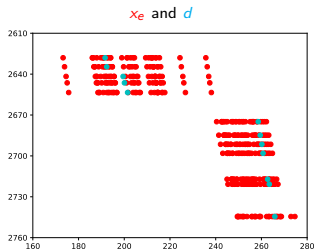


$M()$ and $M() \pm S()$



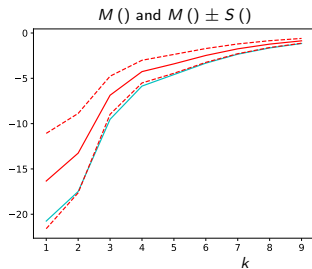
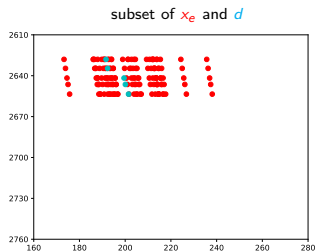
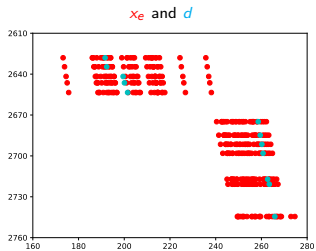
Application to real data

RFT data - results for well F4



Application to real data

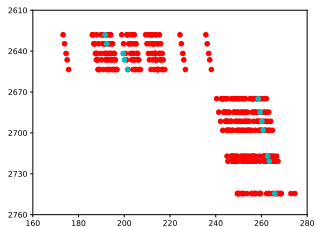
RFT data - results for well F4



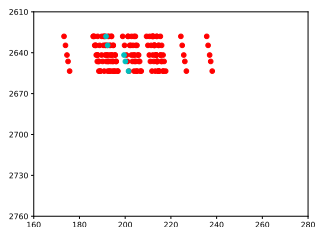
Application to real data

RFT data - results for well F4

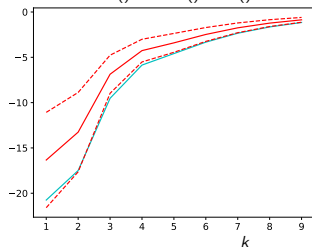
x_e and d



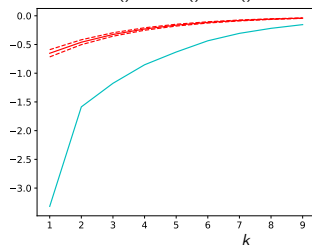
subset of x_e and d



$M()$ and $M() \pm S()$



$M()$ and $M() \pm S()$



Summary

Multiscale model diagnostic (MMD) discriminates well between realizations from different distributions

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MMD is straightforward and computationally inexpensive

Summary

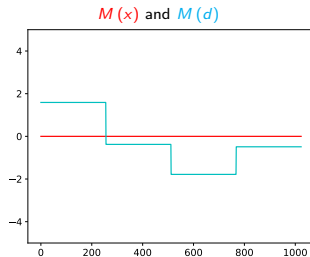
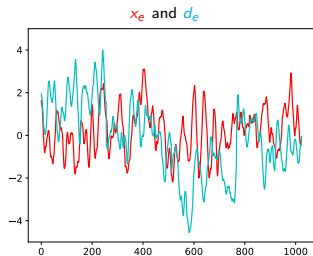
Multiscale model diagnostic (MMD) discriminates well between realizations from different distributions

MMD is straightforward and computationally inexpensive

In simplistic situations, MMD gives guidance regarding what changes that are desirable for the prior predictive distribution

Method assessment on toy problems

Example 5 - blockwise varying data mean



Method assessment on toy problems

Example 5 - blockwise varying data mean

