Multiscale Model Diagnostic

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Model diagnostic of prior predictive distribution

\[ x_e = g(m_e) + \epsilon_e; \quad e = 1, \ldots, E \]

- \( g \): forward model/simulator
- \( m_e \): prior parameter realization
- \( \epsilon_e \): error realization
- \( x_e \): (perturbed) prior prediction
Model diagnostic of prior predictive distribution

... to reduce the risk of unsuccessful data assimilation

Assess if prior predictions are consistent with observed data, that is, if the observed data vector could be a credible realization from the prior predictive distribution
Model diagnostic of prior predictive distribution

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If not consistent, (part of) the modeling setup should be changed before data assimilation is performed
Model diagnostic of prior predictive distribution

...to reduce the risk of unsuccessful data assimilation

Assess if prior predictions are consistent with observed data, that is, if the observed data vector could be a credible realization from the prior predictive distribution

If not consistent, (part of) the modeling setup should be changed before data assimilation is performed

Coverage of individual observations by the ensemble of prior predictions is not sufficient for consistency, as coverage does not take into account trends and shapes
Could the black curve be a credible realization from the prior predictive distribution?
Multiscale model diagnostic of prior predictive distribution

Compare prior predictions and data on multiple scales of variation
Multiscale model diagnostic of prior predictive distribution

Compare prior predictions and data on multiple scales of variation

The method utilizes scalar products of prior predictions and data with certain multiscale vectors

\[ \chi_{k,e} = h_k^T x_e \]
\[ \delta_k = h_k^T d \]

\( x_e \) : (recall) prior prediction
\( d \) : data
\( h_k \) : multiscale vector on scale \( k \)
Multiscale model diagnostic of prior predictive distribution

Multiscale vectors (the four vectors with the longest characteristic length)
With real data, there is only a single data vector available

\[ \delta_k = h_k^T d \]
With real data, there is only a single data vector available

\[ \delta_k = h_k^T d \]

When assessing the method's applicability and robustness on toy problems, I will consider an ensemble of data realizations to avoid effects associated with a particular data vector

\[ \delta_{k,e} = h_k^T d_e \]

Empirical means, \( M(\delta_k) \), and standard deviations, \( S(\delta_k) \), will then be available for comparison with \( M(\chi_k) \) and \( S(\chi_k) \)
Multiscale model diagnostic of prior predictive distribution

Method assessment on toy problems
Method assessment on toy problems

Plot explanation

\[ M(\chi_k) \pm S(\chi_k) \]
Method assessment on toy problems

Plot explanation

\[ x_e \]

\[ M(\chi_k) \text{ and } M(\chi_k) \pm S(\chi_k) \]

\[ \downarrow \rightarrow \leftarrow \]

Multiply \( x_e \) with \( h_0 \) to obtain \( \chi_{0,e} \)

\[ h_0 \]

\[ \text{to obtain } \chi_{0,e} \]
Method assessment on toy problems

Plot explanation

\[ x_e \]

**Multiply** \( x_e \) with \( h_0 \) to obtain \( \chi_{0,e} \)

\[
\downarrow
\]

Repeat for all \( e \) and compute \( M(\chi_0) \) and \( S(\chi_0) \).
Method assessment on toy problems

Plot explanation

$M(\chi_k)$ and $M(\chi_k) \pm S(\chi_k)$

Multiply $x_e$ with $h_0$ to obtain $\chi_{0,e}$

Repeat for all $e$ and compute $M(\chi_0)$ and $S(\chi_0)$
Method assessment on toy problems

Plot explanation

\[ x_e \]

\[ \begin{align*}
M(x_k) \text{ and } M(x_k) \pm S(x_k)
\end{align*} \]

\[ \begin{align*}
\text{multiply } x_e \text{ with } h_1 \\
\text{to obtain } \chi_{1,e}
\end{align*} \]

\[ \downarrow \]

\[ \begin{align*}
\text{Repeat for all } e \text{ and compute } \\
M(\chi_1) \text{ and } S(\chi_1)
\end{align*} \]
Method assessment on toy problems

Plot explanation

\[ M(\chi_k) \text{ and } M(\chi_k) \pm S(\chi_k) \]
Method assessment on toy problems

Plot explanation, use of colors: prior predictive, data

\[ x_e \text{ and } d_e \]

\[ M() \text{ and } M() \pm S() \]
Method assessment on toy problems

Example 1 - $x_e$ and $d_e$ from the same distribution

[Graph showing $x_e$ and $d_e$]

[Graph showing $M()$ and $M() \pm S()$]
Method assessment on toy problems

Example 1 - $x_e$ and $d_e$ from the same distribution

$x_e$ and $d_e$

$M()$ and $M() \pm S()$

$S(x_k)$ and $S(\delta_k)$
Method assessment on toy problems

Example 1 - $x_e$ and $d_e$ from the same distribution with ensemble size 1000

$x_e$ and $d_e$

$M()$ and $M() \pm S()$

$S(x_k)$ and $S(\delta_k)$
Method assessment on toy problems

Example 2

$\mathbf{x}_e$ and $d_e$

$\mathbf{M}(\mathbf{x})$ and $\mathbf{M}(\mathbf{d}) \pm \mathbf{S}(\mathbf{k})$
Method assessment on toy problems

Example 2 - different data means

\[ M(x) \text{ and } M(d) \]

\[ x_e \text{ and } d_e \]

\[ M() \text{ and } M() \pm S() \]
Method assessment on toy problems

Example 2 - different data means

$x_e$ and $d_e$

$M(x)$ and $M(d)$

$M()$ and $M()\pm S()$ ($E = 1000$)

$M()$ and $M()\pm S()$
Method assessment on toy problems

Example 2 - simplified explanation of behaviour

\[ h_0^T M(d) \]
Method assessment on toy problems

Example 2 - simplified explanation of behaviour

\[ h_0^T M(d) > 0 \]
Method assessment on toy problems

Example 2 - simplified explanation of behaviour

\[ h_1^T M(d) \]
Method assessment on toy problems
Example 2 - simplified explanation of behaviour
Method assessment on toy problems

Example 2 - simplified explanation of behaviour

\[ h_1^T M(d) \] ?
Method assessment on toy problems

Example 2 - simplified explanation of behaviour

\[ h_1^T M(d) = 0 \]
Method assessment on toy problems

Example 2 - simplified explanation of behaviour

\[ h_2^T M(d) = 0 \]

\[ M(d) = 0 + - + - \]

\[ h_0 \]

\[ h_1 \]

\[ h_2 \]
Method assessment on toy problems

Example 3

$x_e$ and $d_e$

$M()$ and $M() \pm S()$

$M()$ and $M() \pm S() \ (E = 1000)$
Method assessment on toy problems

Example 3 - decreasing data mean

$x_e$ and $d_e$

$M(x)$ and $M(d)$

$M()$ and $M() \pm S()$

$M()$ and $M() \pm S()$ ($E = 1000$)
Method assessment on toy problems

Example 3 - simplified explanation of behaviour

\[ h_0^T M(d) \]
Method assessment on toy problems

Example 3 - simplified explanation of behaviour

$h_0^T M(d)$ ?
Method assessment on toy problems

Example 3 - simplified explanation of behaviour
Method assessment on toy problems

Example 3 - simplified explanation of behaviour

\[ h_0^T M(d) = 0 \]
Method assessment on toy problems

Example 3 - simplified explanation of behaviour

\[ h_1^T M(d) \]

\[ M(d) \]

\[ h_0 \]

\[ h_1 \]

\[ h_2 \]
Example 3 - simplified explanation of behaviour

\[ h_1^T M(d) \]
Method assessment on toy problems

Example 3 - simplified explanation of behaviour

\[ h_1^T M(d) \]
Method assessment on toy problems

Example 3 - simplified explanation of behaviour

\[ h_1^T M(d) > 0 \]
Method assessment on toy problems

Example 3 - simplified explanation of behaviour

\[ h_2^T M(d) \]
Method assessment on toy problems

Example 3 - simplified explanation of behaviour

\[ M(d) \]

\[ h_2^T M(d) ? \]
Method assessment on toy problems

Example 3 - simplified explanation of behaviour

\[ h_2^T M(d) \]
Method assessment on toy problems

Example 3 - simplified explanation of behaviour

\[ h_2^T M(d) \]
Method assessment on toy problems

Example 3 - simplified explanation of behaviour

\[ h_2^T M(d) > 0 \]

\[ M(d) \]

\[ h_0 \]

\[ h_1 \]

\[ h_2 \]
Method assessment on toy problems

Example 4 - correlation lengths 25 ($x$) and 50 ($d$)
Method assessment on toy problems

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Example 4 - correlation lengths 25 \((x)\) and 50 \((d)\)
Method assessment on toy problems
Example 4 - simplified explanation of behaviour

Let $\nu$ denote either $x$ or $d$
Method assessment on toy problems
Example 4 - simplified explanation of behaviour

Let $\nu$ denote either $x$ or $d$

\[h_k^T v_e\quad ?\]
Method assessment on toy problems
Example 4 - simplified explanation of behaviour

Let $\nu$ denote either $x$ or $d$.

$h_k^T \nu_e$ ?

detail of a potential $\nu_e$

one period of $h_k$
Let $\nu$ denote either $x$ or $d$.
Method assessment on toy problems
Example 4 - simplified explanation of behaviour

Let \( \nu \) denote either \( x \) or \( d \)

\[
h_k^T \nu_e > 0
\]
Method assessment on toy problems

Example 4 - simplified explanation of behaviour

Let \( h_k^T v_e \)?

- Large standard deviation of \( h_k^T v_e \) when characteristic lengths of \( v_e \) and \( h_k \) are similar

Detail of another potential \( v_e \)
Method assessment on toy problems

Example 4 - simplified explanation of behaviour

Let $v$ denote either $x$ or $d$

0

Large standard deviation of $h_k^T v_e$ when characteristic lengths of $v_e$ and $h_k$ are similar
Method assessment on toy problems

Example 4 - simplified explanation of behaviour

\[ h_k^T v_e < 0 \]

Large standard deviation of \( h_k^T v_e \) when characteristic lengths of \( v_e \) and \( h_k \) are similar
Method assessment on toy problems

Example 4 - correlation lengths 25 ($x$) and 50 ($d$)
Method assessment on toy problems

Example 4 - simplified explanation of behaviour

Let $v$ denote either $x$ or $d$

Small standard deviation when characteristic length of $v$ is larger than that of $h_k$

$h_k^T v_e$ ?

detail of potential $v_e$

one period of $h_k$
Method assessment on toy problems

Example 4 - simplified explanation of behaviour

Let $v$ denote either $x$ or $d$.

detail of potential $v_e$

Small standard deviation when characteristic length of $v_e$ is larger than that of $h_k$.

$h_k^T v_e$ ?

one period of $h_k$
Method assessment on toy problems

Example 4 - simplified explanation of behaviour

Let $v$ denote either $x$ or $d$.

Small standard deviation when characteristic length of $v$ is larger than that of $h$. 

$h_k^T v_e$ ?

0

one period of $h_k$

detail of potential $v_e$
Method assessment on toy problems

Example 4 - simplified explanation of behaviour

\[ h_k^T v_e \approx 0 \]

Detail of potential \( v_e \)

One period of \( h_k \)
Method assessment on toy problems

Example 4 - simplified explanation of behaviour

\[ h_k^T \nu_e \approx 0 \]

Small standard deviation when characteristic length of \( \nu_e \) is larger than that of \( h_k \)
Multiscale model diagnostic of prior predictive distribution

Application to real data
Application to real data

The Norne field

[Diagram of Norway showing Voring Basin and More Basin with labels for different areas (C, D, E, G).]

Simulation model geometry

[Map of simulated model geometry with axes X and Y.]
Application to real data

The Norne field

Data from G segment
Application to real data

The Norne field

Data from G segment

Production data from well E4 (wells: black dots on right figure)
Application to real data
The Norne field

Data from G segment
Production data from well E4 (wells: black dots on right figure)
Time-lapse (4-D) impedance data from most of the segment
Application to real data
The Norne field

Data from G segment
Production data from well E4 (wells: black dots on right figure)
Time-lapse (4-D) impedance data from most of the segment
RFT data from wells E4 and F4
Application to real data
Production data - results for well E4

Gas

$x_e$ and $d$
Application to real data
Production data - results for well E4

Gas

$xe$ and $d$

$M()$ and $M() \pm S()$
Application to real data
Production data - results for well E4

Gas

Water

$M()$ and $M() \pm S()$
Application to real data
Production data - results for well E4

Gas

Water
Application to real data
Production data - results for well E4

Water

subset of $x_e$ and $d$

rate (m$^3$/day)

$M()$ and $M() \pm S()$

Water

$x_e$ and $d$

rate (m$^3$/day)

$M()$ and $M() \pm S()$
Application to real data

Time-lapse impedance data

Time-lapse data are obtained by subtracting data acquired from two surveys over the same study region at different times. The aim is to infer fluid movements and/or pressure changes in the subsurface over this time span, and also to infer flow-related rock properties, such as permeability (fluid conductivity).
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Impedance (density × velocity) data for a subsurface region are obtained by inverting seismic data observed in the sea water or at the sea floor. Hence, they are not really data, but it is common to split the assimilation of time-lapse seismic data into flow-related rock properties this way.
Application to real data

Time-lapse impedance data - study region

The major part of the Norne G segment constitutes the study region.
Application to real data

Time-lapse impedance data - multiscale vectors

\[ h_0 \]

\[ h_1 \]

\[ h_2 \]

\[ h_3 \]

\[ h_4 \]
Application to real data

Time-lapse impedance data - time span 2001-2003
Application to real data

Time-lapse impedance data - time span 2001-2003
Application to real data

Time-lapse impedance data - time span 2001-2003 - results

\[ M() \] and \[ M() \pm S() \]
Application to real data
Time-lapse impedance data - time span 2001-2004 - results

\[ M() \text{ and } M() \pm S() \]
Application to real data
Time-lapse impedance data - time span 2001-2006 - results
Application to real data

RFT data

RFT (Repeat Formation Tester) data consist of pressure values along the wellbore

![Well E4 diagram]
RFT (Repeat Formation Tester) data consist of pressure values along the wellbore
Application to real data
RFT data - results for well E4

$x_e$ and $d$

$M()$ and $M() \pm S()$
Application to real data

RFT data - results for well E4

\[ x_\phi \text{ and } d \]

\[ M() \text{ and } M() \pm S() \]
Application to real data
RFT data - results for well E4

$x_e$ and $d$

subset of $x_e$ and $d$

$M()$ and $M() \pm S()$
Application to real data
RFT data - results for well E4

\[ M() \text{ and } M() \pm S() \]

subset of \[ x_e \text{ and } d \]
Application to real data
RFT data - results for well F4
Application to real data
RFT data - results for well F4

$x_e$ and $d$

subset of $x_e$ and $d$

$M()$ and $M() \pm S()$
Application to real data
RFT data - results for well F4

$x_e$ and $d$

subset of $x_e$ and $d$

$M()$ and $M() \pm S()$
Multiscale model diagnostic (MMD) discriminates well between realizations from different distributions
Summary

Multiscale model diagnostic (MMD) discriminates well between realizations from different distributions

MMD is straightforward and computationally inexpensive
Summary

Multiscale model diagnostic (MMD) discriminates well between realizations from different distributions

MMD is straightforward and computationally inexpensive

In simplistic situations, MMD gives guidance regarding what changes that are desireable for the prior predictive distribution
Method assessment on toy problems

Example 5 - blockwise varying data mean

$x_e$ and $d_e$

$M(x)$ and $M(d)$
Method assessment on toy problems

Example 5 - blockwise varying data mean

\[ x_e \text{ and } d_e \]

\[ M(x) \text{ and } M(d) \]

\[ M() \text{ and } M() \pm S() \]

\[ M() \text{ and } M() \pm S() \ (E = 1000) \]